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FLY ROD RESPONSE

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1. INTRODUCTION

The first mode natural frequency, stiffness and deflection profile can be used to describe the performance of a fly rod. An unloaded rod responds at its natural frequency; rods with higher natural frequencies are described by anglers as rods with faster response. Rod stiffness is a measure of its resistance to tip loads which produce bending. Generally stiffer rods are chosen when angling for larger fish. Rods with low butt flex and high tip flex have been considered by anglers as rods with high action. In 1961, the American Fishing Tackle Manufacturers Association set a standard for flylines based on the weight of the first 30 feet of the flyline (Cairns [1]). Today, fly rods are typically labeled with a line weight rating indicating the recommended line weight for use with that rod.

Several recent studies have investigated fly rod performance. Spolek [2] showed a 40% variation in natural frequency values and a 88% variation in stiffness values for 9 ft–6 weight fly rod blanks from different manufacturers. Spolek [3] proposed that manufacturers use a rating system which describes the stiffness and natural frequency of fly rods. Considering rods of a given length and line weight rating, Spolek [4] found that rods with higher natural frequencies have higher line speeds and may be best for long casts, whereas rods with lower natural frequencies may be best for delicate presentations. In a photographic analysis, Robson [5] found mathematical representations for the motion of the butt of a fly rod during a cast. Hoffmann and Hooper [6] developed a numerical model which predicts the performance of rod blanks as a function of the design parameters. They also present an empirical correlation which relates the line weight rating of a fly rod to the stiffness and frequency of the rod.

In this study, a semi-empirical equation is presented which can be used to predict the first mode natural frequency of fly rods and fly rod blanks as a function of the stiffness to mass ratio and a mass distribution parameter. The equation may be applicable to other multi-piece tapered cantilever beams, examples of which include flag poles and antennas.

2. EXPERIMENTAL SYSTEM

Rod stiffness, the first mode natural frequency and rod tip section mass were obtained for five rod blanks and 19 fly rods. A detailed description of the test procedure has been presented by Hooper [7].

Rod stiffness and natural frequency values were obtained by placing the rod butts horizontally in a 28 cm clamp (see Figure 1). For rods with handles longer than 28 cm, overlap occurred on the butt end of the rod so keep the rod handle portion of the rod rigid. The clamp was adjusted to keep the rod butt and tip at the same elevation. Tip loads were applied and a third order polynomial equation of the P–y_t curve was used to obtain

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the tip load required to obtain a tip deflection of $L'/4$ and rod stiffness. To obtain the natural frequency, the rod was tapped to obtain a small deflection vibration. The use of a photoelectric cell which responded to reflection from a light beam aimed at the rod tip, connected to an oscilloscope allowed measurement of the natural frequency of the rod.

Tip section mass was obtained with the use of a digital laboratory balance. Values of x_{cg} were obtained by moving the rod tip section(s) on a fulcrum until the balance point was obtained. The distance from the rod tip to the balance point and the length of the tip section(s) were measured with a tape measure.

Uncertainties for the measurements, obtained using the method of Kline and McClintock [8] with 20:1 odds, are: stiffness $\pm 3\%$, natural frequency $\pm 5\%$ and mass $\pm 1\%$.

3. GOVERNING EQUATIONS

Mark's equation [9] for the first mode natural frequency of a tapered cantilever beam and an equation for the deflection of a cantilever beam with an applied tip load are presented below. The equations are valid for small tip deflections; the parameters c_1, c_2, c_3, c_4, and C are dependent upon the mass distribution of the rod:

$$f = c_1\sqrt{EIg/L'^3/L'A\gamma}, \qquad y_t = PL'^3/c_3EI, \tag{1, 2}$$

where $L'A\gamma = M'c_2$. If the rod stiffness (S) is defined as P_t/y_t when $y_t = L'/4$, then

$$S = c_3EI/L'^3. \tag{3}$$

Combining equations (1) and (3), the first mode natural frequency is shown to be a function of the mass distribution parameter and the stiffness to mass ratio of the rod,

$$f = c_4\sqrt{S/M'}. \tag{4}$$

In this study, equation (5) is used in place of equation (4) because the mass of the tip section(s) is easy to obtain for rods and rod blanks without rod destruction:

$$f\,(\mathrm{Hz}) = C\sqrt{S\,(\mathrm{g/cm})/M_t\,(\mathrm{g})}. \tag{5}$$

Barten [10], Parthap and Varadan [11], Takahashi [12], Verma and Murthy [13] and Hoffmann and Hooper [6] all found that the natural frequency of cantilever beams was essentially independent of amplitude of tip deflection for deflections less than $L'/4$.

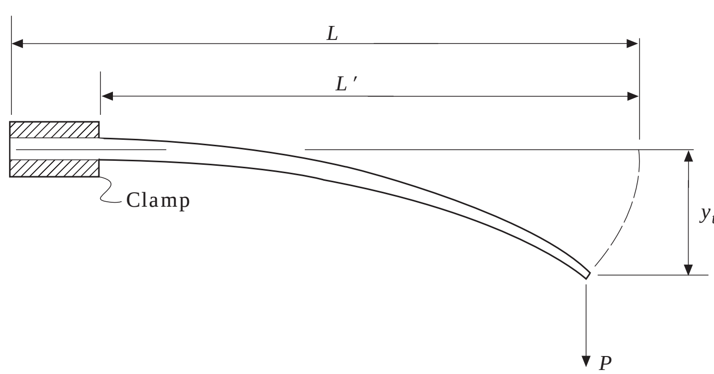

Figure 1. Experimental system.

TABLE 1

measured data

Rod	Length (ft)	Wt (No. of pieces)	S (g/cm)	f (Hz)	M_t (g)	x_{cg}/L_t	$C\sqrt{S/M_t}$
(a) Fly rods							
Berkley HRX40	8′6″	6–7(2)	1·11	2·33	18·0	0·596	2·31
Lamiglas G1298	9′	7(2)	1·04	1·92	20·6	0·559	1·85
Lamiglas GF108	9′	4–5(2)	0·95	2·06	19·0	0·609	2·16
Lamiglas XT905	9′	5(4)	1·08	2·52	15·9	0·601	2·58
G. Loomis FR114741MX	9′6″	7(4)	1·33	2·88	16·7	0·602	2·81
Orvis Gr. Mt. Ser.	9′	9(2)	1·55	2·50	22·1	0·611	2·62
Orvis Rock. Mt. Ser.	9′	4(2)	0·98	2·43	16·8	0·642	2·57
Orvis Trident	9′	5(2)	1·05	2·55	18·6	0·635	2·48
Orvis PM10	9′	9(2)	2·02	3·13	23·5	0·644	3·12
Orvis PM10	9′	11(4)	2·79	3·53	27·4	0·640	3·52
Sage B490GFL*	9′	4(2)	0·78	2·26	14·3	0·600	2·20
Sage III 470LL	7′	4(2)	0·91	3·30	7·15	0·585	3·19
Sage III 896RPL	9′6″	8(2)	1·36	2·89	15·8	0·616	2·90
Sage III 379-3LL	7′9″	3(3)	0·79	2·91	14·6	0·615	2·90
Sage III 990-3RPLX	9′	9(3)	1·83	3·23	33·1	0·646	3·14
Sage IV 590-3SP	9′	5(3)	1·09	2·76	25·1	0·639	2·74
Shakespeare FY1100	8′6″	8(2)	1·10	2·02	25·1	0·602	1·98
South Bend G-685	8′6″	6–7(2)	1·03	2·81	11·6	0·600	2·80
R. L. Winston IM6	8′6″	3(3)	0·69	2·67	14·9	0·602	2·60
(b) Rod blanks							
Fenwick HMG GFL	9′	6(2)	1·00	3·03	11·1	0·631	3·09
Fenwick HMG GFL	9′	10(2)	1·99	3·35	17·5	0·609	3·26
Fenwick HMG GFL	7′6″	5(2)	0·84	3·24	8·1	0·645	3·45
Lamiglas HSGF	9′	6(2)	1·31	4·00	10·2	0·639	3·78
Lamiglas HSGF	7′6″	5(2)	1·31	4·28	7·1	0·617	4·18

* Constructed using a commercial blank

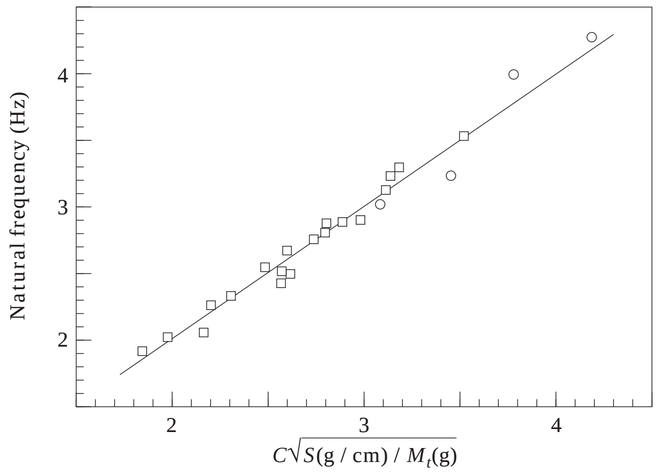

Figure 2. Natural frequency correlation: ○, rod blanks; □, rods.

4. PREDICTION OF NATURAL FREQUENCY

Data for 19 graphite fly rods and 5 graphite rod blanks is presented in Table 1. In equation (5), the mass distribution parameter was found to vary as much as $\pm 11\%$ for two, three or four piece rods, and this variation was found to correlate with the location of the center of gravity of the tip section(s) of the rods and rod blanks. Rods with more massive ferrules have larger values of C. Equation (5) was found to predict the natural frequency of both fly rods and rod blanks within $\pm 6\%$, close to the uncertainty of the measurements, with the relationships for C presented below.

For the case of two piece rods, $C = 28{\cdot}7(x_{cg}/L_t) - 7{\cdot}8$; M_t, x_{cg} and L_t were obtained for the tip section of the rod. For the case of three piece rods, $C = 28{\cdot}7(x_{cg}/L_t) - 5{\cdot}2$; M_t, x_{cg} and L_t were obtained for the top two sections of the rod. For the case of four piece rods, $C = 28{\cdot}7(x_{cg}/L_t) - 7{\cdot}3$; M_t, x_{cg} and L_t were obtained for the top two sections of the rod.

A curve of the natural frequency versus $C\sqrt{S/M_t}$ is presented in Figure 2. Equation (5) is a generalized semi-empirical equation and is independent of rod length, rod material, line weight rating and rod manufacturer. It allows the consumer and manufacturer to evaluate the response of fly rods with simple tests to evaluate stiffness, tip section mass and tip section center of gravity.

For the case of uniform area cantilever beams, the relationships presented above for two, three and four piece rods were used to obtain values of $C = 6{\cdot}55$, $9{\cdot}15$ and $7{\cdot}05$, respectively, using $x_{cg}L_t = 0{\cdot}5$. Corresponding values of $C = 7{\cdot}10$, $8{\cdot}30$ and $7{\cdot}10$ were calculated using equations 1, 3, and 5 with $L_t = 0{\cdot}55L'$ for two and four piece rods, $L_t = 0{\cdot}75L'$ for three piece rods, a value of $c_1 = 0{\cdot}56$ obtained from Marks [9], a value of $c_2 = 1$ for a constant area rod, a value of $c_3 = 3$ for low deflection tip loaded cantilever beams, and a value of S 11·5% larger than the low deflection stiffness (obtained numerically by Hooper [7] for a constant area rod). Although the relationships for C were fitted to data for fly rods, the calculated values of C for uniform area rods are within 9% of the relationships presented above.

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NOMENCLATURE

A	cross-sectional area of rod at beginning of flexible portion of rod
C, c	parameters dependent upon the mass distribution of the rod
E	modulus of elasticity of rod material
f	first mode natural frequency of rod
g	gravitational constant
I	moment of inertia of rod at beginning of flexible portion of rod
L	rod length
L'	length of flexible portion of rod or rod blank
M'	mass of flexible portion of rod or rod blank
M_t	mass of tip section(s) of rod or rod blank
P	static tip force
P_t	static tip force to obtain a rod tip deflection of $L'/4$
S	rod stiffness parameter $= P_t/(L'/4)$
x_{cg}	distance from rod tip to center of gravity of tip section(s)
y	rod deflection
γ	specific weight of rod material

Subscript

t	Rod tip location or rod tip section